We would like to submit this paper for consideration for the best paper award.

HAMILTONIAN DECOMPOSITION OF THE AUGMENTED CUBES

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ABSTRACT

The n-dimensional hypercube network Q_n is one of the most studied interconnection networks due to its rich combinatorial structure and symmetry. The problem of decomposing a hypercube into edge-disjoint Hamiltonian cycles was first posed by G. Ringel in 1956. He proved that the $n = 2^m$-dimensional hypercube, for $m \geq 1$, admits a Hamiltonian decomposition and posed the question of whether any n-dimensional hypercube possesses such a decomposition. Later, Aubert and Schneider (1982) established the existence of Hamiltonian decompositions for all n. In 2004, Bae presented a constructive approach for generating these cycles in hypercubes.

The n-dimensional augmented cube AQ_n, introduced by Choudum, is an important variation of the hypercube that exhibits several properties superior to those of the standard hypercube. Although other variants of the hypercube have the same number of vertices and edges as Q_n, the augmented cube is notable because it maintains the same number of vertices but nearly doubles the number of edges. This property makes AQ_n a compelling candidate for studying Hamiltonian decompositions. Hung (2012) raised the question of whether AQ_n admits an edge-disjoint Hamiltonian decomposition.

In this paper, we leverage the algebraic structure of AQ_n as a Cayley graph to define AQ_n using refined approach. We address the open problem of Hamiltonion decomposition for AQ_n by introducing a novel construction based on Gray codes. Specifically, with the help of this construction and coding theoretic approach, we demonstrate how to decompose an augmented cube into edge-disjoint hamiltonion cycles. We prove that it admits $n - 1$ such cycles for any n. Thereby confirmimg a conjecture by Hung (2012).